

Math tricks

Study guide · 8 topics

Divisibility rules

Tell whether a number divides evenly without doing the division. They make factoring, simplifying fractions, and spotting primes much faster.

Divisible by	When	Example
2	The last digit is even: 0, 2, 4, 6, or 8.	3,578 : ends in 8
3	The digits add up to a multiple of 3.	4,281 : $4 + 2 + 8 + 1 = 15$, and 15 is a multiple of 3
4	The last two digits make a multiple of 4.	7,316 : 16 is a multiple of 4
5	The last digit is 0 or 5.	2,345 : ends in 5
6	It passes the tests for both 2 and 3.	1,314 : even, and $1 + 3 + 1 + 4 = 9$
7	Double the last digit and subtract it from the rest. If what is left is a multiple of 7 (0 counts), so is the number. Repeat for big numbers.	343 : $34 - 2 \times 3 = 28$, and $28 = 4 \times 7$
8	The last three digits make a multiple of 8.	5,096 : $096 = 96 = 12 \times 8$
9	The digits add up to a multiple of 9.	6,237 : $6 + 2 + 3 + 7 = 18$
10	The last digit is 0.	4,590 : ends in 0
11	Alternately add and subtract the digits from the left. If the result is a multiple of 11 (0 counts), so is the number.	2,728 : $2 - 7 + 2 - 8 = -11$
12	It passes the tests for both 3 and 4.	1,932 : $1 + 9 + 3 + 2 = 15$, and 32 is a multiple of 4

To check whether a number is prime, you only need to try primes up to its square root. For 91: $\sqrt{91}$ is a bit under 10, so try 2, 3, 5, 7. And $91 = 7 \times 13$, so it is not prime.

Mental math shortcuts

Most arithmetic on the test has a faster route than writing it out.

Times 5

Multiply by 10, then halve.

$$68 \times 5 \rightarrow 680 \div 2 = \mathbf{340}$$

Times 25

Multiply by 100, then divide by 4.

$$36 \times 25 \rightarrow 3,600 \div 4 = \mathbf{900}$$

Times 9

Multiply by 10, then take away the number once.

$$47 \times 9 \rightarrow 470 - 47 = \mathbf{423}$$

Times 11, two digits

Spread the digits apart and put their sum in the middle. If the sum is 10 or more, carry the 1.

$$53 \times 11 \rightarrow 5 _ 3 \text{ with } 5 + 3 = 8 \text{ in the middle} = \mathbf{583}$$

Squaring a number ending in 5

Multiply the front part by one more than itself, then write 25 after it.

$$65^2 \rightarrow 6 \times 7 = 42, \text{ then } 25 = 4,225$$

Two numbers the same distance from a round one

Use $(a + b)(a - b) = a^2 - b^2$.

$$48 \times 52 \rightarrow 50^2 - 2^2 = 2,500 - 4 = 2,496$$

Halve one, double the other

The product stays the same. Keep going until it is easy.

$$16 \times 35 \rightarrow 8 \times 70 = 560$$

Percents swap

$x\%$ of y is the same as $y\%$ of x . Pick whichever is easier.

$$8\% \text{ of } 50 \rightarrow 50\% \text{ of } 8 = 4$$

Build a percent from 10% and 1%

10% moves the decimal one place left, 1% moves it two.

Add up the pieces.

$$15\% \text{ of } 240 \rightarrow 10\% \text{ is } 24, 5\% \text{ is } 12, \text{ together } 36 = 36$$

Numbers worth knowing by heart

Recognizing these on sight turns a lot of questions into one step.

Squares

$$1^2 = 1 \quad 2^2 = 4 \quad 3^2 = 9 \quad 4^2 = 16 \quad 5^2 = 25 \quad 6^2 = 36 \quad 7^2 = 49 \quad 8^2 = 64 \quad 9^2 = 81 \quad 10^2 = 100$$

$$11^2 = 121 \quad 12^2 = 144 \quad 13^2 = 169 \quad 14^2 = 196 \quad 15^2 = 225 \quad 16^2 = 256 \quad 17^2 = 289 \quad 18^2 = 324 \quad 19^2 = 361$$

$$20^2 = 400 \quad 21^2 = 441 \quad 22^2 = 484 \quad 23^2 = 529 \quad 24^2 = 576 \quad 25^2 = 625$$

Cubes

$$1^3 = 1 \quad 2^3 = 8 \quad 3^3 = 27 \quad 4^3 = 64 \quad 5^3 = 125 \quad 6^3 = 216 \quad 7^3 = 343 \quad 8^3 = 512 \quad 9^3 = 729 \quad 10^3 = 1,000$$

Powers of 2

$$2^0 = 1 \quad 2^1 = 2 \quad 2^2 = 4 \quad 2^3 = 8 \quad 2^4 = 16 \quad 2^5 = 32 \quad 2^6 = 64 \quad 2^7 = 128 \quad 2^8 = 256 \quad 2^9 = 512$$

$$2^{10} = 1,024 \quad 2^{11} = 2,048 \quad 2^{12} = 4,096$$

Primes below 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

There are 25 of them. 1 is not prime, and 2 is the only even prime.

Fractions, decimals, and percents

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	$0.\overline{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.\overline{6}$	$66\frac{2}{3}\%$
$\frac{1}{4}$	0.25	25%

$\frac{3}{4}$	0.75	75%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%
$\frac{1}{6}$	$0.1\overline{6}$	$16\frac{2}{3}\%$
$\frac{5}{6}$	$0.8\overline{3}$	$83\frac{1}{3}\%$
$\frac{1}{8}$	0.125	$12\frac{1}{2}\%$
$\frac{3}{8}$	0.375	$37\frac{1}{2}\%$
$\frac{5}{8}$	0.625	$62\frac{1}{2}\%$
$\frac{7}{8}$	0.875	$87\frac{1}{2}\%$
$\frac{1}{9}$	$0.\overline{1}$	$11\frac{1}{9}\%$
$\frac{1}{10}$	0.1	10%
$\frac{1}{12}$	$0.08\overline{3}$	$8\frac{1}{3}\%$

A bar means the digits under it repeat forever. Eighths go up by 0.125 each: $1/8 = 0.125$, $2/8 = 0.25$, $3/8 = 0.375$, and so on.

Exponents and roots

Rule	Example
Multiply the same base: add the exponents. $a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^4 = x^7$
Divide the same base: subtract them. $a^m \div a^n = a^{m-n}$	$y^8 \div y^3 = y^5$
A power of a power: multiply them. $(a^m)^n = a^{mn}$	$(2^3)^2 = 2^6 = 64$
Anything (except 0) to the 0 is 1.	$7^0 = 1$

A negative exponent flips it under 1.

$$a^{-n} = \frac{1}{a^n} \qquad 5^{-2} = \frac{1}{25}$$

A fraction exponent is a root. $a^{1/n}$ is the n th root; $a^{m/n}$ is that root to the m .

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$$

A negative base: an even power is positive, an odd power negative.

$$(-3)^2 = 9, \text{ but } -3^2 = -9$$

Watch the parentheses.

Simplifying a square root

Pull out the biggest perfect square that divides it: $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$. Roots multiply and divide freely, $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$, but they do not add: $\sqrt{9} + \sqrt{16} = 7$, while $\sqrt{25} = 5$.

Estimating a root

Find the squares on either side: $\sqrt{50}$ is between $\sqrt{49} = 7$ and $\sqrt{64} = 8$, and much closer to 7 (it is about 7.07).

Scientific notation

Move the decimal point until one digit is left in front, and count the moves: $45,000 = 4.5 \times 10^4$, and $0.0032 = 3.2 \times 10^{-3}$. To multiply, multiply the fronts and add the exponents.

Algebra

Patterns to recognize

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(x + 3)^2 = x^2 + 6x + 9$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(x - 5)^2 = x^2 - 10x + 25$$

$$(a + b)(a - b) = a^2 - b^2$$

$$x^2 - 49 = (x + 7)(x - 7)$$

Factoring $x^2 + bx + c$

Find two numbers that multiply to c and add to b . For $x^2 + 7x + 12$: $3 \times 4 = 12$ and $3 + 4 = 7$, so it is $(x + 3)(x + 4)$. If c is negative, the two numbers have opposite signs.

Multiplying two binomials (FOIL)

First, Outer, Inner, Last: $(x + 2)(x - 5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$.

Solving

Whatever you do to one side, do to the other. When you multiply or divide an inequality by a negative number, flip the sign: $-2x > 6$ becomes $x < -3$. For two equations, add or subtract them so one letter cancels.

Lines

Slope is rise over run: $m = \frac{y_2 - y_1}{x_2 - x_1}$. In $y = mx + b$, m is the slope and b is where the line crosses the y -axis.

Written as $Ax + By = C$, the slope is $-\frac{A}{B}$.

Geometry

Right triangles with whole sides

3, 4, 5 6, 8, 10 9, 12, 15 5, 12, 13 8, 15, 17 7, 24, 25 20, 21, 29

Any multiple of one works too. If two sides are 15 and 20, the third is probably 25 ($5 \times$ the 3-4-5 triangle). Otherwise, $a^2 + b^2 = c^2$, where c is the longest side.

Special right triangles

45-45-90: sides $x, x, x\sqrt{2}$. **30-60-90:** sides $x, x\sqrt{3}, 2x$, where the shortest side faces the 30° angle.

Formulas

Rectangle	area = length $\times$ width, perimeter = $2(\text{length} + \text{width})$
Triangle	area = $\frac{1}{2} \times \text{base} \times \text{height}$; the angles add to 180°
Circle	area = πr^2 , circumference = $2\pi r = \pi d$
Trapezoid	area = $\frac{1}{2} \times (\text{base}_1 + \text{base}_2) \times \text{height}$
Box	volume = length $\times$ width $\times$ height
Cylinder	volume = $\pi r^2 h$
Any polygon	the inside angles add to $(n - 2) \times 180^\circ$, where n is the number of sides
Distance between points	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, the Pythagorean theorem in disguise

A circle that fits exactly inside a square has a diameter equal to the side of the square. Doubling every side of a shape multiplies its area by 4 and a solid's volume by 8.

Word problem shortcuts

Arithmetic Reasoning is mostly a handful of setups in different clothes.

Distance, rate, time	distance = rate $\times$ time. Cover the one you want: rate = distance $\div$ time.
Working together	If one job takes a hours and another b hours, together it takes $\frac{a \times b}{a + b}$. Two pipes that fill a tank in 3 and 6 hours: $18 \div 9 = 2$ hours.
Percent change	$\frac{\text{change}}{\text{original}}$. From 40 to 50 is up $10 \div 40 = 25\%$; from 50 back to 40 is down $10 \div 50 = 20\%$.
Two discounts in a row	Multiply what is left: 20% off then 10% off leaves $0.8 \times 0.9 = 0.72$, which is 28% off, not 30%.
Ratios	Add the parts. Split 40 in the ratio 3 : 5: 8 parts, 5 each, so 15 and 25.
Averages	total = average $\times$ count. To find a missing score, work out the total it needs and subtract what you have.

Simple interest	interest = principal $\times$ rate $\times$ time. \$500 at 4% for 3 years: $500 \times 0.04 \times 3 = \$60$.
Before a percent change	If \$63 is the price after 10% off, it is 90% of the original: $63 \div 0.9 = \$70$.

Test strategy

- **Answer every question.** A blank counts the same as a wrong answer, and there is no extra penalty for guessing, so never leave one empty.
- **Know your pace.** Math Knowledge gives you about 53 seconds a question (25 in 22 minutes); Arithmetic Reasoning about 70 (25 in 29). If one is eating your time, make your best guess and move on.
- **Work backward.** The answer is one of the five choices. Plug them in, starting from the middle one, and see which works.
- **Use easy numbers.** When a question is all letters, pick small values (like $x = 2$), work out the answer, and see which choice matches.
- **Estimate first.** A rough answer often rules out three choices before you do any real work.
- **Check the units and the question.** Many wrong choices are the right number for a different question: the width instead of the area, the discount instead of the sale price.