

Fundamentals

Study guide · 10 topics

Adding

Line the numbers up by place value (ones under ones, tens under tens) and add from the right. When a column comes to 10 or more, write the ones digit and carry the tens to the next column.

$$\begin{array}{r} 11 \\ 487 \\ + 365 \\ \hline 852 \end{array}$$

$7 + 5 = 12$: write 2, carry 1. $1 + 8 + 6 = 15$: write 5, carry 1. $1 + 4 + 3 = 8$.

Adding a column

With three or more numbers, look for pairs that make 10 or 100 first. $38 + 47 + 62$ is $(38 + 62) + 47 = 100 + 47 = 147$.

Check it: subtract one number from the answer and you should get the other. $852 - 365 = 487$.

Subtracting

Line up by place value and subtract from the right. When the top digit is smaller, borrow 1 from the next column over, which adds 10 to this one.

$$\begin{array}{r} 691 \\ 703 \\ - 268 \\ \hline 435 \end{array}$$

3 is less than 8, and the tens column has 0 to borrow from, so borrow from the hundreds: 7 becomes 6, the 0 becomes 10, then 9 after lending 1 to the ones, and the 3 becomes 13. Now $13 - 8 = 5$, $9 - 6 = 3$, and $6 - 2 = 4$.

Counting up

Across zeros it is often easier to count up from the smaller number. For $1,000 - 368$: 368 to 400 is 32, 400 to 1,000 is 600, so the answer is 632.

Check it: add the answer to the number you took away. $435 + 268 = 703$.

Multiplying

Know the times table through 12 cold; everything else is built from it.

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

The highlighted diagonal is the perfect squares. The table is symmetric: 7×8 is the same as 8×7 , so there are only half as many facts to learn as it looks.

Bigger numbers: break one apart

Split one number into tens and ones, multiply each part, and add: $46 \times 23 = 46 \times 20 + 46 \times 3 = 920 + 138 = 1,058$. This is exactly what the written method does, one row per digit.

Zeros

Multiply the nonzero parts, then tack on all the zeros: $300 \times 40 = 12$ followed by three zeros = 12,000. Anything times 0 is 0; anything times 1 is itself.

Dividing

Long division repeats four steps from the left: divide, multiply, subtract, bring down.

$$\begin{array}{r}
 144 \\
 6 \overline{) 864} \\
 \underline{6} \\
 26 \\
 \underline{24} \\
 24 \\
 \underline{24} \\
 0
 \end{array}$$

6 goes into 8 once ($1 \times 6 = 6$, leaving 2). Bring down the 6: 26. 6 goes into 26 four times (24, leaving 2). Bring down the 4: 24, which is exactly four 6s. So $864 \div 6 = 144$.

If something is left at the end, that is the remainder: $50 \div 7 = 7$ remainder 1, or $7\frac{1}{7}$.

Dividing by a decimal

Move the decimal point in both numbers until you are dividing by a whole number. $4.5 \div 0.15$ is the same as $450 \div 15 = 30$.

Check it: multiply back. $144 \times 6 = 864$.

Negative numbers

Adding a negative is subtracting	$5 + (-8) = 5 - 8 = -3$
Subtracting a negative is adding	$4 - (-6) = 4 + 6 = 10$
Same signs multiply or divide to positive	$(-3) \times (-7) = 21$, and $(-20) \div (-4) = 5$
Different signs give negative	$(-3) \times 7 = -21$, and $20 \div (-4) = -5$

Think of a number line: adding moves right, subtracting moves left, and a negative sign turns you around. An even count of negatives multiplies to positive; an odd count, negative.

Order of operations

Work in this order: **parentheses**, then **exponents**, then **multiplication and division** together from left to right, then **addition and subtraction** together from left to right. (PEMDAS, but M and D are a tie, and so are A and S.)

$$\begin{aligned}
 &8 + 2 \times (7 - 4)^2 \div 6 \\
 &= 8 + 2 \times 3^2 \div 6 \\
 &= 8 + 2 \times 9 \div 6 \\
 &= 8 + 18 \div 6 \\
 &= 8 + 3 = 11
 \end{aligned}$$

The usual trap: $12 \div 2 \times 3$ is 18 (left to right), not 2.

Fractions

Simplify	Divide the top and bottom by the same number: $\frac{18}{24} = \frac{3}{4}$ (both $\div 6$).
Add or subtract	Get a common bottom first: $\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$.
Multiply	Straight across: $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$. Cancel first to keep numbers small.
Divide	Flip the second and multiply: $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$.
Mixed numbers	$2\frac{3}{5} = \frac{2 \times 5 + 3}{5} = \frac{13}{5}$. Turn them into this form before multiplying or dividing.

Compare

Cross-multiply: $\frac{3}{7}$ vs $\frac{2}{5} \rightarrow 3 \times 5 = 15$ vs $2 \times 7 = 14$, so $\frac{3}{7}$ is bigger.

Decimals

Add or subtract

Line up the decimal points (fill with zeros): $12.5 + 3.75 = 12.50 + 3.75 = 16.25$.

Multiply

Multiply as whole numbers, then count the decimal places in both: $0.3 \times 0.04 \rightarrow 3 \times 4 = 12$, three places $\rightarrow 0.012$.

Divide

Move the decimal in both numbers until the one you divide by is whole: $7.2 \div 0.08 = 720 \div 8 = 90$.

Times or divided by 10, 100, 1,000

Move the point one place per zero: $3.6 \times 100 = 360$; $3.6 \div 100 = 0.036$.

Place value

In 4,827.395: 4 thousands, 8 hundreds, 2 tens, 7 ones, 3 tenths, 9 hundredths, 5 thousandths.

Percents

Percent means "out of 100": $35\% = \frac{35}{100} = 0.35$. To find a percent of something, turn the percent into a decimal and multiply: $35\% \text{ of } 80 = 0.35 \times 80 = 28$.

Part of a whole

part = percent $\times$ whole. $15\% \text{ of } 60 = 0.15 \times 60 = 9$.

What percent?

percent = part $\div$ whole. $18 \text{ out of } 24 = 0.75 = 75\%$.

Find the whole

whole = part $\div$ percent. $12 \text{ is } 30\% \text{ of what? } 12 \div 0.3 = 40$.

Percent to decimal

Move the point two places left: $7\% = 0.07$, $125\% = 1.25$.

More shortcuts are under Math tricks.

Rounding and estimating

To round, look at the digit just to the right of where you are rounding: 5 or more rounds up, 4 or less rounds down. 3,746 to the nearest hundred is 3,700; 3,756 is 3,800.

Estimate before you calculate. 49×21 is about $50 \times 20 = 1,000$, so an answer of 102.9 or 10,290 is clearly wrong. On a multiple-choice test, an estimate often picks the answer by itself.